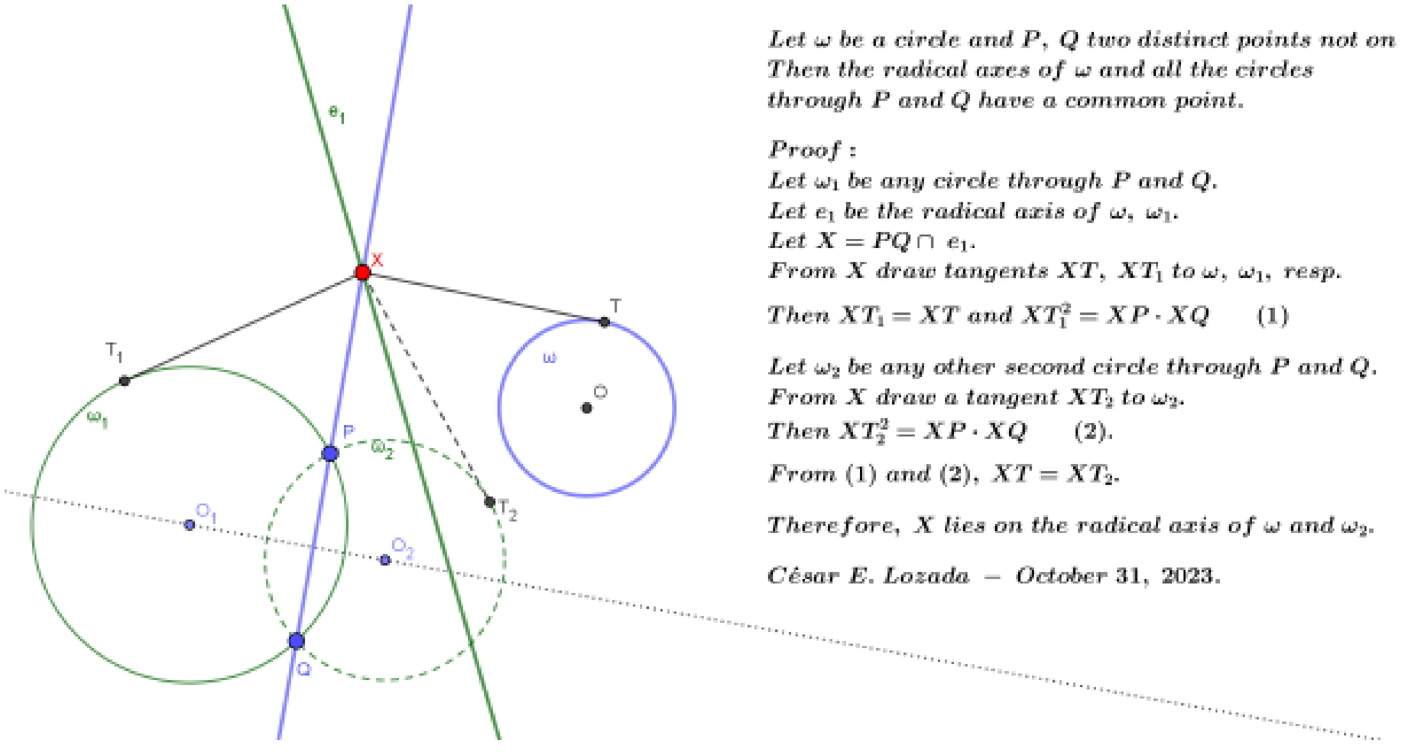

Let ω be a circle and P, Q two distinct points not on ω. Then the radical axes of ω and all the circles through P and Q have a common point.

Proof :
Let ω_1 be any circle through P and Q.
Let e_1 be the radical axis of ω, ω_1.
Let $X = PQ \cap e_1$.
From X draw tangents XT, XT_1 to ω, ω_1, resp.

Then $XT_1 = XT$ and $XT_1^2 = XP \cdot XQ$ (1)

Let ω_2 be any other second circle through P and Q.
From X draw a tangent XT_2 to ω_2.
Then $XT_2^2 = XP \cdot XQ$ (2).

From (1) and (2), $XT = XT_2$.

Therefore, X lies on the radical axis of ω and ω_2.

César E. Lozada – October 31, 2023.