Theorem. Consider $\triangle ABC$ on the plane with circumcenter $O$. $P$ is a point on the plane, not lying on the lines $OA, OB, OC$. Let $\triangle A_1B_1C_1$ be the pedal triangle of a point $P$ with respect to $\triangle ABC$. $A_2$ is the point, other than $A$, that circles $(ABC)$ and $(AB_1C_1)$ intersect and define $B_2, C_2$ cyclically. $A_3 = BB_2 \cap CC_2$ and define $B_3, C_3$ cyclically. Let $\triangle A_4B_4C_4$ be the circumcevian triangle of $P$ with respect to $\triangle ABC$. Then:

1. $\triangle A_2B_2C_2$ and $\triangle A_4B_4C_4$ are perspective.

2. $\triangle A_3B_3C_3$ and $\triangle A_4B_4C_4$ are perspective.