Theorem. Let $\triangle A_1B_1C_1$ be the circumcevian triangle of a point $P$ with respect to $\triangle ABC$. Let $H_{bc}, H_{cb}, H_{ac}, H_{ab}, H_{ba}$ be respectively the orthocenter of $\triangle PB_1C, \triangle PC_1B, \triangle PA_1A, \triangle PA_1C, \triangle PA_1B, \triangle PB_1A$.

Let $A' = H_{ca}H_{ac} \cap H_{ab}H_{ba}$, $B' = H_{ab}H_{ba} \cap H_{bc}H_{cb}$, $C' = H_{bc}H_{cb} \cap H_{ca}H_{ac}$.

Then

- Three lines $AA', BB', CC'$ are concurrent at point $X$.
- Three lines $A_1A', B_1B', C_1C'$ are concurrent at point $X'$.
- $P, X, X'$ are collinear.