1. Each of the following statements has one of the forms

\[ \sim p \quad p \land q \quad p \lor q \quad p \rightarrow q \quad p \leftrightarrow q \]

Find the appropriate form and indicate what each statement variable in your choice represents.

(a) If Tobias passes the audition, then he will not cry.
   Ans: \( p \rightarrow q \)

(b) Freemasons do not run the country.
   Ans: \( \sim p \)

(c) Fluffy is happy if and only if he is lying in the sun.
   Ans: \( p \leftrightarrow q \)

(d) Either I will get her flowers or she will hurt me.
   Ans: \( p \lor q \)

(e) Mac has a concussion but Charlie does not.
   Ans: \( p \land q \)

2. Write the negation of each statement in the previous problem.
   Ans:
   (a) Tobias passes the audition and he will cry.
   (b) Freemasons run the country.
   (c) Either Fluffy is lying in the sun and he is not happy, or Fluffy is not lying in the sun and he is happy.
   (d) I will not get her flowers and she will not hurt me.
   (e) Mac does not have a concussion or Charlie does.

3. Use a truth table to determine if the following logical equivalence holds:

\[ (p \land q) \rightarrow r \equiv p \rightarrow (\sim q \lor r) \]

Ans: the statements are logically equivalent.

4. Use truth tables to determine whether the following arguments are valid or invalid:

(a) \[ p \rightarrow (q \land r) \]
   \[ \sim r \]
   \[ \therefore \sim p \]
   Ans: this is valid
(b) \[ p \land q \]
\[ p \rightarrow r \]
\[ q \rightarrow r \]
\[ \therefore \sim r \]

Ans: this is invalid

5. Write an argument form for each of the arguments below, then decide whether the argument is valid or invalid.

(a) Either there are snakes on the plane, or someone just said, “What?”.
   If there are snakes on the plane, then Samuel L. Jackson is upset.
   If someone just said, “What?”, then Samuel L. Jackson is upset.
   Therefore, Samuel L. Jackson is upset.

Ans: Let \( p \) be “there are snakes on the plane”, let \( q \) be “someone just said, “What?””, and let \( r \) be “Samuel L. Jackson is upset”.
Then, the argument form is
\[ p \lor q \]
\[ p \rightarrow r \]
\[ q \rightarrow r \]
\[ \therefore r \]
and one may check that this is a valid argument form.

(b) This real number is rational or it is irrational.
   This real number is not rational.
   Therefore, this real number is irrational.

Ans: Let \( p \) be “this real number is rational” and let \( q \) be “this real number is irrational”.
Then, the argument form is
\[ p \lor q \]
\[ \sim p \]
\[ \therefore q \]
and again one may check that this is a valid argument form.

6. Let \( \mathbb{Z}^+ \) be the set of positive integers, and let \( \mathbb{R}^+ \) be the set of positive real numbers. Indicate which of the following statements are true and which are false.

(a) \( \forall x \in \mathbb{Z}^+, \exists y \in \mathbb{Z}^+ \) such that \( x = y + 1 \)
   Ans: false (take \( x = 1 \))

(b) \( \forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} \) such that \( x = y + 1 \)
   Ans: true (take \( y = x - 1 \))

(c) \( \exists x \in \mathbb{R} \) such that \( \forall y \in \mathbb{R}, x = y + 1 \)
   Ans: false

(d) \( \forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+ \) such that \( xy = 1 \)
   Ans: true (\( y = \frac{1}{x} \in \mathbb{R}^+ \))
(e) \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ such that } xy = 1 \)
    Ans: false (take \( y = 0 \))

(f) \( \forall x \in \mathbb{Z}^+ \text{ and } \forall y \in \mathbb{Z}^+, \exists z \in \mathbb{Z}^+ \text{ such that } z = x - y \)
    Ans: false (take \( x = y = 1 \))

(g) \( \forall x \in \mathbb{Z} \text{ and } \forall y \in \mathbb{Z}, \exists z \in \mathbb{Z}^+ \text{ such that } z = x - y \)
    Ans: false (take \( x = y = 1 \))

(h) \( \exists u \in \mathbb{R}^+ \text{ such that } \forall v \in \mathbb{R}^+, uv < v \)
    Ans: true (take \( u = \frac{1}{2} \))

7. Write the negation of each statement in the previous problem. Which of the negations are true?
   Ans:
   (a) \( \exists x \in \mathbb{Z}^+ \text{ such that } \forall y \in \mathbb{Z}^+, x \neq y + 1 \) true
   (b) \( \exists x \in \mathbb{Z} \text{ such that } \forall y \in \mathbb{Z}, x \neq y + 1 \) false
   (c) \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ such that } x \neq y + 1 \) true
   (d) \( \exists x \in \mathbb{R}^+ \text{ such that } \forall y \in \mathbb{R}^+, xy \neq 1 \) false
   (e) \( \exists x \in \mathbb{R} \text{ such that } \forall y \in \mathbb{R}, xy \neq 1 \) true
   (f) \( \exists x, y \in \mathbb{Z}^+ \text{ such that } \forall x \in \mathbb{Z}^+, x \neq x - y \) true
   (g) \( \exists x, y \in \mathbb{Z} \text{ such that } \forall z \in \mathbb{Z}^+, z \neq x - y \) true
   (h) \( \forall u \in \mathbb{R}^+, \exists v \in \mathbb{R}^+ \text{ such that } uv \geq v \) false

8. Write an argument form for each of the arguments below, then decide whether the argument is valid or invalid. Define any domains, predicates, or other statement variables that you use.
   (a) If an infinite series converges, then its terms go to 0.
      The terms of the infinite series \( \sum_{n=1}^{\infty} \frac{n}{n+1} \) do not go to 0.
      \[ \therefore \text{ The infinite series } \sum_{n=1}^{\infty} \frac{n}{n+1} \text{ does not converge.} \]
      Ans: Let \( D = \{ \text{all infinite series} \} \), let \( P(x) \) be “\( x \) converges”, and let \( Q(x) \) be “the terms of \( x \) go to 0”.
      Then, the argument form is
      \[ \forall x \in D, \text{ if } P(x) \text{ then } Q(x). \]
      \[ \sim Q\left( \sum_{n=1}^{\infty} \frac{n}{n+1} \right) \]
      \[ \therefore \sim P\left( \sum_{n=1}^{\infty} \frac{n}{n+1} \right) \]
      This argument form is valid by Universal Modus Tollens.

   (b) Any sum of two rational numbers is rational.
      If I stick this extra line in here, then no one will notice.
      The sum \( r + s \) is rational.
      No one noticed.
      \[ \therefore \text{ The numbers } r \text{ and } s \text{ are rational.} \]
Ans: The relevant parts of this argument are the first, third, and last statements; the other ones can be ignored. We need to get the first statement into if-then form. We can do this by letting $D = \mathbb{R}$, $P(x, y)$ be “$x$ and $y$ are rational”, and letting $Q(x)$ be “$x$ is rational”. The relevant part of the argument is:

$\forall x \in D$, if $P(x, y)$ then $Q(x + y)$.

$Q(r + s)$

$\therefore P(r + s)$

This argument form is invalid; it exhibits the converse error.

(c) Nothing intelligible ever puzzles me.

Logic puzzles me.

$\therefore$ Logic is unintelligible.

Ans: The first statement means the same thing as, “If something is intelligible, then it does not puzzle me.” So, let $P(x)$ be “$x$ is intelligible” and let $Q(x)$ be “$x$ does not puzzle me”. Then, the argument form is

$\forall x \in D$, if $P(x)$ then $Q(x)$.

$\sim Q(\text{Logic})$

$\therefore \sim P(\text{Logic})$

This argument form is valid by Universal Modus Tollens.