Math 491: Partial Differential Equations
Fall 2008

Each problem is worth 10 points, for a total of forty. Be sure to justify your claims and explain your reasoning. This assignment is due on Monday, December 8.

1. Use the method of characteristics to solve the problem.

\[
PDE \quad u_t + 3t^2 u_x = 2tu \quad -\infty < x < \infty, \quad 0 < t < \infty \\
IC \quad u(x,0) = \sin 2x \quad -\infty < x < \infty
\]

2. (Lesson 27, problem 5) Use the method of characteristics to solve the problem.

\[
PDE \quad u_x + 2u_y + 2u = 0 \quad -\infty < x < \infty, \quad -\infty < y < \infty \\
Initial curve \quad u(x,y) = F(x,y) \quad \text{on the curve } x = y
\]

3. Verify the transform of the three-dimensional Laplacian

\[
\nabla^2 u = u_{xx} + u_{yy} + u_{zz}
\]

into spherical coordinates.

4. A spherical shell with an inner radius 1 and an outer radius 2 has a steady-state temperature distribution. Its inner boundary is held at 100°C. Its outer boundary satisfies \( \frac{\partial u}{\partial \rho} = -\gamma \), where \( \gamma > 0 \) is constant.

a. Find the temperature \( u \). (Hint: The temperature depends only on the radius, \( \rho \).)

b. What are the hottest and coldest temperatures? Why?

c. Find \( \gamma \) so that the temperature on its outer boundary is 20°C.