This assignment is worth 50 points, distributed as indicated. Be sure to justify your claims and explain your reasoning. This assignment is due on **Wednesday, January 21**.

1. (An alternative proof of Theorem 1.4) Suppose $S$ is a nonempty subset of the real numbers that is bounded below. Let $L$ be the set of all lower bounds of $S$. Then $\alpha = \sup L$ exists and $\alpha = \inf S$. In particular, $\inf S$ exists. [7 pts]

2. Prove or disprove the statement: If $S$ and $T$ are nonempty, bounded subsets of real numbers, then

$$
\sup(S \cup T) = \max\{\sup S, \sup T\}. \quad [7 \text{ pts}]
$$

3. Let $n$ be a natural number. Use induction to show that all numbers of the form $7^n - 2^n$ are divisible by 5. [7 pts]

4. (§1.1#15) For a set of numbers $S$, a member $c$ of $S$ is called the *maximum* of $S$ provided that it is an upper bound of $S$. Prove that a set $S$ of numbers has a maximum if and only if it is bounded above and $\sup S$ belongs to $S$. Give an example of a set $S$ of numbers that is bounded above, but has no maximum. [7 pts]

5. Find the maximum, minimum, infimum and supremum of the set, if they exist. [4 pts each]
   a. \{1 - \frac{1}{3^n} | n \in \mathbb{N}\}
   b. \{n + \frac{(-1)^n}{n} | n \in \mathbb{N}\}

6. (§1.3#6) Which of the following inequalities hold for all real numbers $a$ and $b$? Justify your conclusions. [3 pts each]
   a. $|a + b| \geq |a| + |b|$
   b. $|a + b| \leq |a| - |b|$

7. (§1.3#7) By writing $a = (a + b) + (-b)$, use the triangle inequality to obtain $|a| - |b| \leq |a + b|$. Then interchange $a$ and $b$ to show that

$$
||a| - |b|| \leq |a + b|.
$$

Then replace $b$ by $-b$ to obtain

$$
||a| - |b|| \leq |a - b|. \quad [8 \text{ pts}]
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