Math 324
Final Review (Comprehesive)
Spring 2008

Name: Key

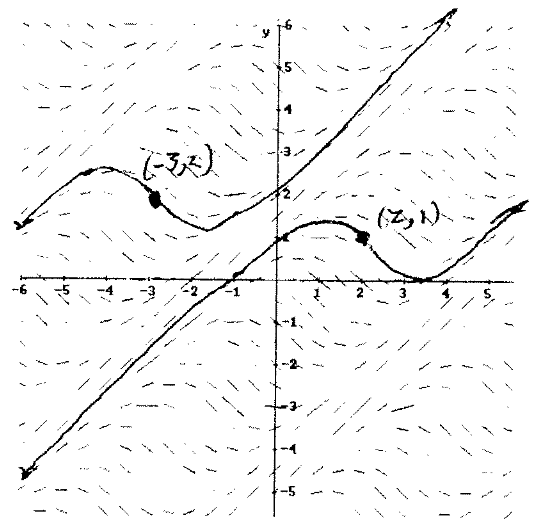
Date: ____________

This review sheet is worth ten points. It is due on Friday, May 2. **The final is May 2 at 10:15 AM in KC 100**

1. Sketch an approximate solution curve that passes through the indicated points.

a. $y(-3) = 2$

b. $y(2) = 1$



2. Solve the exact equation $\left(1 + \ln x + \frac{y}{x}\right)\, dx = (1 - \ln x)\, dy$.

$$f(x,y) = (x+y)\ln x - y = c$$

3. Solve the IVP.

a. $x'' + 4x' + 5x = \delta(t - \pi) + \delta(t - 2\pi)$, $x(0) = 0$ and $x'(0) = 2$.

$$x(t) = e^{-2(t-\pi)}\sin(t-\pi)\,\mathcal{U}(t-\pi) + e^{-2(t-2\pi)}\sin(t-2\pi)\,\mathcal{U}(t-2\pi) + 2e^{-2t}\sin t$$
$$= -e^{-2(t-\pi)}\sin t\,\mathcal{U}(t-\pi) + e^{-2(t-2\pi)}\sin t\,\mathcal{U}(t-2\pi) + 2e^{-2t}\sin t$$

b. $y'' - 2y' + 2y = e^{-t}$, $y(0) = 0$ and $y'(0) = 1$.

$$y(t) = \frac{1}{5}e^{-t}\cancel{\frac{7}{5}}e^{t}\cos t + \frac{7}{5}e^{t}\sin t$$
$$-\frac{1}{5}$$

c. $y'' + 4y = f(t)$, where $f(t) = \begin{cases} \sin t & 0 \le t < \pi \\ 2\sin t & t \ge \pi, \end{cases}$ $y(0) = 0$ and $y'(0) = 0$.

$$y(t) = \frac{1}{3}\sin t - \frac{1}{6}\sin 2t - \mathcal{U}(t-\pi)\left[\frac{1}{3}\sin(t-\pi) - \frac{1}{6}\sin(t-\pi)\right]$$
$$= \frac{1}{3}\sin t - \frac{1}{6}\sin 2t + \mathcal{U}(t-\pi)\left[\frac{1}{3}\sin t - \frac{1}{6}\sin t\right]$$

4. Find a general solution of the ODE.

a. $y''' + 7y'' + 10y' = 0$

$$y(x) = C_1 + C_2 e^{-2x} + C_3 e^{-5x}$$

b. $3y'' + 2y' + 2y = 0$

$$y(x) = e^{-\frac{1}{3}x}\left(C_1 \sin\frac{\sqrt{5}}{3}x + C_2 \cos\frac{\sqrt{5}}{3}x\right)$$

c. $y'' + 16y = \sin 4x$

$$y(x) = C_1 \sin 4x + C_2 \cos 4x - \frac{1}{8}x\cos 4x$$

d. $y'' + 9y = 9\sec^2 3x$, where $0 < x < \pi/6$

$$y = C_1 \sin 3x + C_2 \cos 3x + \ln|\sec 3x + \tan 3x| \sin 3x \,\cancel{+}\, - 1$$

5. Solve the system $x' = x + 2y$, $y' = x + e^{-t}$ for the initial values $x(0) = 0$ and $y(0) = 0$.

$$x(t) = \frac{-2}{9}e^{-t} - \frac{2}{3}te^{-t} + \frac{2}{9}e^{2t}$$

$$y(t) = \frac{\cancel{2}\ 1}{9}e^{-t} \cancel{-\frac{1}{3}} +\frac{2}{3} te^{-t} - \frac{\cancel{2}\ 1}{9}e^{2t}$$