7.1] Show how \( 426 + 38 \) can be performed on an abacus.

7.2] #3 Find the chords of \( 90^\circ \) and \( 120^\circ \).

\[ \text{and } 90^\circ \text{ By assumption} \]
\[ r = 60^\circ. \text{ Thus by the Pythagorean thm} \]
\[ (\text{and } 90^\circ)^2 = 2r^2 = 2(60^\circ)^2 \]
\[ \text{Thus } \text{and } 90^\circ = \sqrt{2(60^\circ)^2} \]
\[ \approx 84^\circ 51' 10'' \]
Since \( \angle CDB = 60^\circ \) and \( \triangle ACDB \) is isosceles, it follows that \( \triangle ACB \) is equilateral.

Thus \( \angle B = 60^\circ \). Since \( \triangle ABC \) is inscribed in a semi-circle, \( \angle ACB \) is right.

By the Pythagorean Theorem,

\[
\text{Area} = \pi r^2 = \frac{1}{2} AB \cdot BC = \frac{1}{2} (2r) r = r^2
\]

Thus

\[
\text{Area} = \pi r^2 = \frac{1}{2} AB \cdot BC = \frac{1}{2} (2r) r = r^2
\]

Hence \( \text{Area} = \pi r^2 = \frac{1}{2} AB \cdot BC = \frac{1}{2} (2r) r = r^2 \)

Area, Circ: Square the diameter, multiply by \( \pi \); divide by 14; divide by 14; 
(\( \frac{11}{14} \)) ; 
Circumference is mult. the diameter by \( \pi \) or divide by \( \pi \).

Area, Triangle & Exact

Area, Triangle & Exact
7.4  
4c  
Solve using Diophantus's techniques:

From two given numbers, subtract the same number so that the remainders have the given ratio. Find the subtracted number. Given numbers 20, 100; given ratio 6:1

\[ \text{Solution} \quad \text{The two remainders are } 20 \text{UM} \text{Mx and } 100 \text{UM} \text{Mx [20-x and 100-x]} \text{. To satisfy the given ratio, we have } 120 \text{UM} \text{Mx is 100UMx [6(20-x) = 120-6x = 100-x]. This } Z0U \text{ is } 5x \text{ [20 = 5x] and the subtracted number is 4} \]

7.4  
5b  
Find two numbers such that the square of either minus the sum of both gives a square.

\[ \text{Solution} \quad \text{Suppose the two numbers are } x+1 \text{ and } 3x+1. \text{ Then } (x+1)^2 - (4x+2) = (x-1)^2. \text{ Suppose that } (3x+1)^2 - (4x+2) = 9x^2. \text{ Then } 9x^2 + 2x - 1 = 9x^2. \text{ It follows that } x = \frac{1}{3}. \text{ Thus these such numbers are } \frac{3}{2} \text{ and } 5 \frac{1}{2}. \]
8.1 Use the rod system to find

437 × 61

8.1 179,712 ÷ 234

1591 

- 6 × 234

768

1872

234

1591 

- 6 × 234

768

1872

234
Modify the technique of Yang Hui to find \( \sqrt[3]{50653} \)

Use 1 fewer column.

<table>
<thead>
<tr>
<th>HF</th>
<th>HL</th>
<th>SL</th>
<th>S</th>
<th>SS</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>30</td>
<td>900</td>
<td>50,653</td>
<td>30</td>
</tr>
<tr>
<td>+30</td>
<td>+1,800</td>
<td>-27,000</td>
<td>7</td>
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</tr>
<tr>
<td></td>
<td>+679</td>
<td>+23,653</td>
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<td>+7</td>
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<tr>
<td>97</td>
<td>x7</td>
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<td></td>
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<td>679</td>
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</tbody>
</table>

So, \( \sqrt[3]{50653} = 37 \)
8.2 #4

Two sheaves of a good harvest and 1 sheet of a mediocre harvest yield 1 ton. Three sheaves of a mediocre harvest and 1 of a poor harvest yield 1 ton. One sheet of a good harvest and 4 of a poor harvest yield 1 ton. How much is a sheet of each?

\[
\begin{align*}
\text{Sol'n} & \quad \text{Good mediocme peel yield} \\
& \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & 4 & 1 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
R_3 &= \frac{\begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & 4 & 1 \end{bmatrix}}{\begin{bmatrix} 2 & 1 \cr 3 & 1 \cr 1 & 4 \end{bmatrix}} \\
R_3 &= \frac{\begin{bmatrix} 0 & 1 \cr 0 & 3 \cr 0 & 0 \end{bmatrix}}{25} \\
&= \begin{bmatrix} \frac{2}{25} & \frac{1}{25} \cr 0 & 1 \cr 0 & 0 \end{bmatrix}
\end{align*}
\]

- 1 sheet of poor yields \( \frac{4}{25} \) ton
- 1 sheet of mediocre yields \( \frac{1}{3} - \frac{4}{25} = \frac{7}{25} \) ton
- 1 sheet of good yields \( \frac{1}{2} - \frac{7}{25} = \frac{9}{25} \) ton
8.3 Use false position: The third part of a necklace of pearls, broken in a lover's quarrel, fell to the ground; its fifth part rested on the couch; the sixth part saved by the wrench; and the tenth part taken by her lover; six pearls remained strong. How many pearls composed the necklace?

\[
\begin{array}{cccc}
\text{Ground} & \frac{1}{3} & 10 \\
\text{Couch} & \frac{1}{5} & 6 \\
\text{Wrench} & \frac{1}{2} & 5 \\
\text{Lover} & \frac{1}{10} & 3 \\
\hline
\text{remaining} & \frac{24}{24} & 6 \rightarrow 6 \text{ remain}
\end{array}
\]

\text{Guess: 30 pearls}