Writing projects for calculus and liberal arts mathematics.

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Why use writing projects in math courses?

- Easy answer: You have to!
- Satisfying answer: They are more fun than homework problems.
- Better answer: To teach students to communicate mathematical ideas.

Warning: Students *will* complain.

Resources

- A. Crannell, G. LaRose, T. Ratliff and E. Rykken, *Writing Projects for Mathematics Courses*, MAA, 2004.
  - Examples for Survey of Math, Calculus and Differential Equations
  - Varying levels of difficulty
  - Complete solutions (some of which can be found online)
- Classroom Capsules, *The College Mathematics Journal*, MAA
  - More difficult (suitable for Calc III or Differential Equations)
  - Not posed as a writing project
- Textbooks

About UE

- Selective liberal arts college with 2500 undergraduate students
- Classes include: Mathematical Ideas, College Algebra, Calculus and Differential Equations
- Average class size (where I have used projects): 30

Example: A Good Project

Dear Math students,

As the new host of America's favorite pricing game, I get asked all the time about the best strategies for playing our games and the chances of winning big prizes. Unfortunately, probability is not one of my strong suits. In a surprising bit of coincidence, I happened to mention this situation in passing to a consultant from International Enterprises, Inc., J.R. Doe, who recommended that I contact you for assistance.

One of the games on The Price is Right is called $\frac{1}{2}$-Off. A contestant has an opportunity to win $10,000 by choosing the correct box from a field of 16. To give the player a better chance to guess the correct box, there are three opportunities to eliminate half of the remaining boxes by winning mini-games. For a complete run-down, check out the premier of the game, as hosted by my legendary predecessor, Bob Barker; Professor Tweddle has gladly posted a link to the YouTube video on his website.

I would like to know the probability of choosing the correct box if there are 16, 8, 4 or 2 boxes remaining when the contestant makes her selection. If there is a 50-50 chance of winning the mini-game to eliminate half the boxes, what is the probability that a contestant would end up with 16, 8, 4 or 2 boxes? Finally, putting all this together, I'd like to know the probability that a contestant wins the game.

Since we will be taking our summer hiatus soon, please respond by Monday, April 26. I look forward to hearing from you.

Sincerely, Drew Carey

Example: A Disappointing Project

Dear Math students,

Thank you so much for your assistance finding the life-time safe lead consumption levels. My boss was pleased with the explanation I was able to provide her supporting my conclusions. This fall, we at the Division of Water Management were confronted with an algae problem in the Ohio River (the source for our local drinking water). In order to remove the algae, we resorted to chemical treatment of the water. This technique had unfortunate side-effects. You may have noticed the unpleasant odor and taste of the tap water in our community this fall.

We are in the process of researching algae removal techniques that will have a lesser impact on the taste and smell of the water. One idea that is looking promising is the use of natural occurring bacteria that consume algae. I could really use your helping developing a mathematical model that will predict the number of bacteria present at a given time. The tricky part is that the bacteria population is influenced by the temperature of the water and the water temperature at our treatment facility depends on the time of day. Tests in the lab have determined that the number bacteria present, N, in thousands per gallon of water is related to the Celsius temperature, C, by the function

$$N(C) = -0.05C^2 + 2.85C + 10, \quad \text{for} \quad 10 \le C \le 40.$$

The temperature (for the summer months) depends upon the time of day, t in hours after midnight, by the function

$$C(t) = -0.0093t^3 + 0.33t^2 - 2.49t + 24, \quad \text{for} \quad 0 \le t < 24.$$

How can I combine these two functions to express the number of bacteria in terms of time? For example, what is the population at 6 AM? at noon? at 6 PM? at midnight? I would also like to know what time of day the bacteria reaches is minimum and maximum population levels. Since the semester is winding down and many of you will be busy with finals and holiday travel, I'd greatly appreciate it if you could respond by Monday, December 6.

Sincerely, Needepe N. Sludge

Most recent example with student solutions

Dear Math students,

I work as a claim adjuster at Regressive Insurance. We have recently run into some difficulty settling a client's claim involving a policy rider that restricts the use of his vehicle on unpaved roadways. According to his policy, he is not to travel more than half a mile on unimproved road surfaces. Last week, this client hit a tree along a twisty, bumpy dirt road, causing damage to his vehicle. According to the report, he was traveling to visit his grandmother when he a moose ran into the road. When he swerved to avoid hitting the animal, he lost control of the vehicle and hit a tree. He claims that the road is less than a half mile. I have been sent out to investigate. Unfortunately, the odometer in my car is broken, so I cannot measure the distance he traveled from the paved road to the accident site directly. However, the speedometer is operational, so with the help of my colleague Rita, I was able to record the car's speed every 10 seconds. I know that distance is related to time and speed, but I cannot figure out how to compute the total distance based on this information. I am hoping that your mathematical expertise can help me settle this. Below is a table of the data I collected.

Time (sec)	Velocity (ft/sec)	Time (sec)	Velocity (ft/sec)
0	0	70	15
...	...	...	...
50	44	120	35
60	35		

In addition to an estimate of the distance he traveled on the dirt road, please provide an explanation of your solution, as I will have to justify any decision made regarding his claim.

Velocity = $\frac{\text{distance}}{\text{time}}$

$\frac{0}{0} = 0$	30 · 10 = 300	22 · 10 = 220	35 · 10 = 350
44 · 10 = 440	44 · 10 440	35 · 10 = 350	
15 · 10 = 150	35 · 10 350	44 · 10 = 440	
35 · 10 350	15 · 10 = 150	30 · 10 = 300	

440
+150
+350
+300
+440
+150
+350
+220
+350
+440
+300
+350

3840 ft / 120 sec

5280 ft/mile

$$\frac{3840 \text{ ft}}{5280 \text{ ft/mile}} = .7\overline{272} \text{ miles}$$

↑ larger than half mile

① I took each velocity and multiplied by 10 seconds to get how many feet she had traveled in 10 seconds.

② I added the totals together to see how many feet she had traveled over all.

③ I took 5280 ft because that is how many there are in a mile. I divided 3840 ft by 5280 to see how many miles she had travelled and it came out to be more than .50 miles which means he should not have been travelling on that road.

MATH 221 PROJECT #2 4/26/2011

TO ACCURITLY APPROXIMATE THE DISTANCE TRAVELED, WE CAN PLOT A GRAPH OF THE VELOCITY VS. TIME ~~GRAPH~~ DATA. BY DOING THIS, WE CAN APPROXIMATE THE AREA UNDER THE CURVE BY USING RIGHT AND LEFT HAND ENDPOINTS AND AVERAGING THE 2 TOGETHER. TO DETERMINE THE TWO, SEE GRAPH 1 FOR LEFT HAND AND GRAPH 2 FOR RIGHT HAND ENDPOINTS. TO FIGURE THE LEFT HAND ENDPOINTS, WE USE THE EQUATION:

$$LH = 10\,(f(0) + f(10) + f(20) + f(30) + f(40) + f(50) + f(60) + f(70) + f(80) + f(90) + f(100) + f(110))$$

WHEN WE SUBSTITUTE VALUES WE GET:

$$LH = 10(0+44+15+35+30+44+35+15+22+35+44+30)$$

WHICH WHEN EVALUATED EQUALS:

$$LH = 3490 \text{ ft.}$$

TO FIND RIGHT HAND ENDPOINTS, WE DO THE SAME, ONLY USING THE RIGHT HAND VALUES. THE EQUATION WE USE IS:

$$RH = 10(f(10) + f(20) + f(30) + f(40) + f(50) + f(60) + f(70) + f(80) + f(90) + f(100) + f(110) + f(120))$$

WHEN SUBSTITUTING, WE GET

$$RH = 10(44+15+35+30+44+35+15+22+35+44+30+35)$$

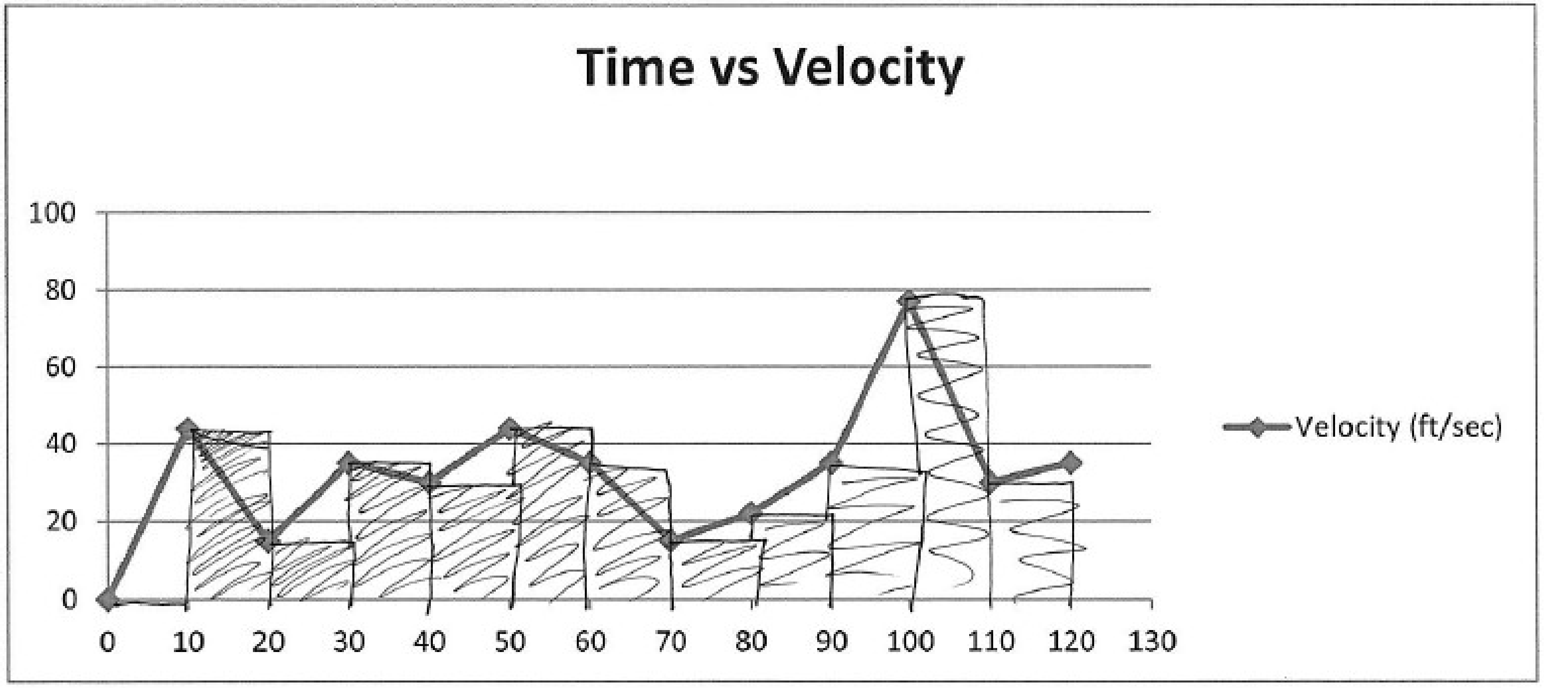
WHICH WHEN EVALUATED EQUALS:

$$RH = 3840 \text{ ft.}$$

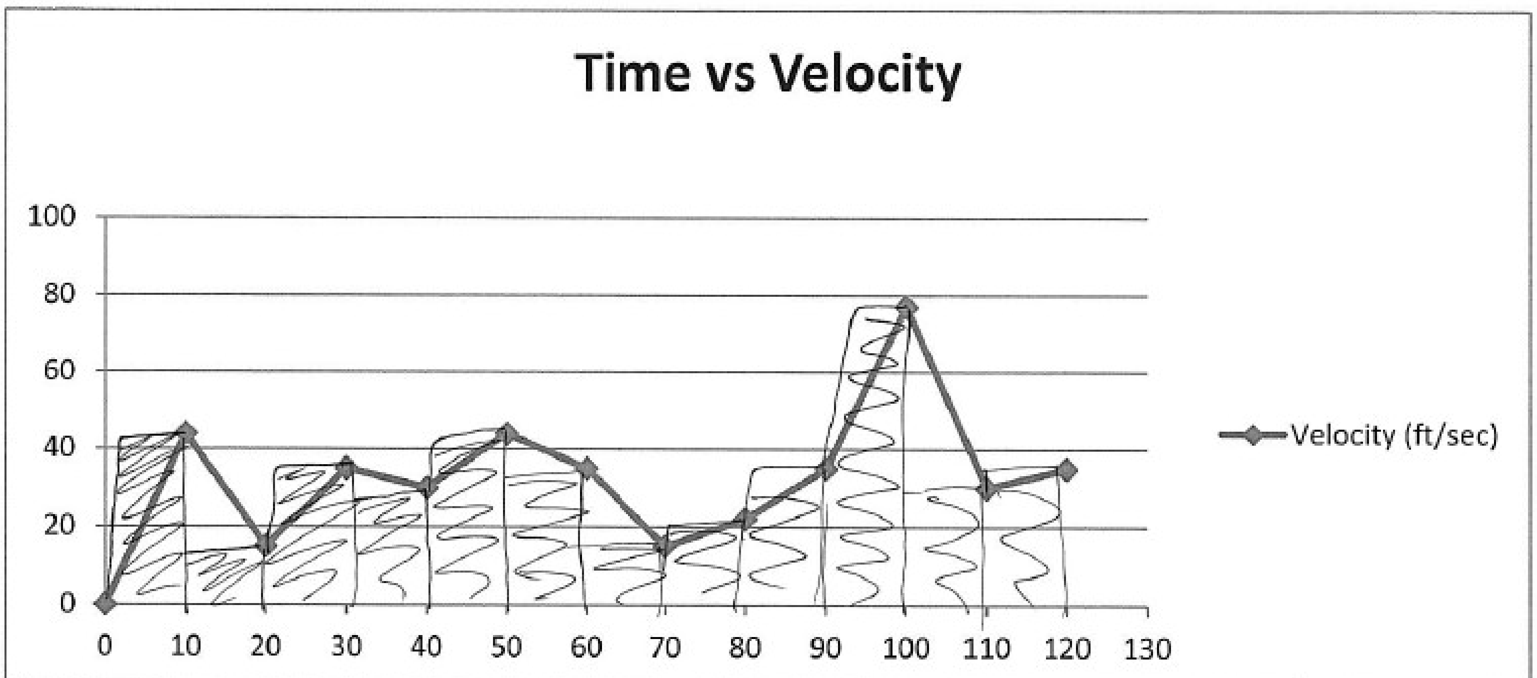
WHEN AVERAGING THESE, WE GET:

$$\frac{3840 + 3490}{2} = 3665 \text{ ft.}$$

SO IN TOTAL, THE ART ROAD IS 3665 FT, WHICH IS MORE THAN THE HALF MILE ALLOCATED IN HIS INSURANCE POLICY.



Time vs Velocity
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
Velocity (ft/sec)



Time vs Velocity
100
80
60
40
20
0
0
10
20
30
40
50
60
70
80
90
100
110
120
130
Velocity (ft/sec)

Rubric

1. Written explanation of the model.

Lacking	0	1	2	3	4	5	Clear

2. Use of proper grammar, style, spelling, etc.

Lacking	0	1	2	3	4	5	Clear

3. Development of mathematical model.

Lacking	0	1	2	3	4	5	Clear

4. Implementation of model.

Lacking	0	1	2	3	4	5	Clear

5. Concise summary of findings; conclusion.

Lacking	0	1	2	3	4	5	Clear

Thank you

Questions?

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