



Given two non – concentric circles  $c_1$  and  $c_2$  and a point  $P$ , neither on any of the given circles nor on their radical axis, there exists a unique circle through  $P$  and coaxial with the given circles.

*Proof :*

Assume  $c$  is the searched circle. Its center lies on the line joining the centers of  $c_1$  and  $c_2$ .

A second point  $P'$  on it is needed for determining  $c$ .

Let  $e$  be the common radical axis of  $c$ ,  $c_1$ ,  $c_2$ .

Take a point  $Q$  on  $e$  and draw the line  $QP$  cutting  $c$  again in  $P'$ .

Let  $QT_1$ ,  $QT_2$ ,  $QT$  be tangents from  $Q$  to  $c_1$ ,  $c_2$ ,  $c$ , respectively.

Then  $QT = QT_1 = QT_2$  and  $QP \cdot QP' = QT^2$

Therefore, it is enough to take  $QP' = QT_1^2/QP$  (signed distances) for determining  $P'$ .

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