Some Problems On Apollonian Gasket

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Abstract

I proposed some problems on Apollonian gasket configuration

Consider $ABC$ be a triangle, construct three circles $(A)$, $(B)$, $(C)$ such that them tangent to each other. Let $(A_1)$ is the circle tangent to the Soddy circle and $(B)$ and $(C)$, let $(A_{k+1})$ is the circle tangent the $(A_k)$ and $(B)$ and $(C)$ for $k = 2, 3, ... n$ define $(B_i)$, $(C_i)$ cyclically. We have some problems in next pages.

![Figure 1]
Problem 1. Three lines $A_j A_k$, $B_j B_k$, $C_j C_k$ are concurrent for any $j \neq k, j, k = 1, 2, ..., n$ (Figure 2).
Problem 2. Three line $AA_k$, $BB_k$, $CC_k$ are concurrent, for $k = 1, 2, ..., n$
Problem 3. Let $(A)$ tangent to $(B_k)$, $(C_k)$ at $A_{ck}$, $A_{bk}$. Define $B_{ck}$, $B_{ak}$, $C_{ak}$, $C_{bk}$ cyclicly. Then six points $A_{bk}$, $A_{ck}$, $B_{ck}$, $B_{ak}$, $C_{ak}$, $C_{bk}$ lie on a circle for $k = 1, 2, \ldots, n$, and the centers of these new circles lie on a line.

Figure 4
Problem 4. Let \((A_k)\) tangent to \((A_{k+1})\) at \(T_{ak}\), define \(T_{ak}, T_{ck}\) cyclically. Then three lines \(T_{aj}T_{ak}, T_{bj}T_{bk}, T_{cj}T_{ck}\) are concurrent for any \(j \neq k, j, k = 1, 2, \ldots, n\).
Problem 5. Circle \( (T_{ak}T_{bk}T_{ck}) \) tangent to six circles \((A_k), (A_{k-1}), (B_k), (B_{k-1}), (C_k), (C_{k-1}) \) any \( k = 1, 2, ..., n \).
Problem 6. Three lines $AT_{ak}, BT_{bk}, CT_{ck}$ are concurrent, for any $k = 1, 2, \ldots, n$. 

Figure 7
Problem 7. Three lines $A_jT_{ak}, B_jT_{bk}, C_jT_{ck}$ are concurrent for any $j, k = 1, 2, ..., n$.

Figure 8

References


[2] [https://artofproblemsolving.com/community/c6h555078p3225247](https://artofproblemsolving.com/community/c6h555078p3225247)

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